INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **37** (2004) 1–8 PII: S0305-4470(04)68157-1

Statistics of equally weighted random paths on a class of self-similar structures

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Received 27 August 2003, in final form 15 October 2003
Published 10 December 2003
Online at stacks.iop.org/JPhysA/37/1 (DOI: 10.1088/0305-4470/37/1/001)

Abstract

We study the statistics of equally weighted random walk paths on a family of Sierpinski gasket lattices whose members are labelled by an integer b ($2 \leqslant b < \infty$). The obtained exact results on the first eight members of this family reveal that, for every $b > 2$, mean path end-to-end distance grows more slowly than any power of its length N. We provide arguments for the emergence of usual power law critical behaviour in the limit $b \to \infty$ when fractal lattices become almost compact.

PACS numbers: 64.60.−i, 05.40.Fb, 64.60.Ak

1. Introduction

The statistics of random walk (RW) paths on self-similar structures has attracted a lot of attention over the last two decades [1]. The main focus of related research activities was on the study of large-scale behaviour of various discrete models. As a result of these studies it was recognized that, in the case of RWs on fractals with coordination number that can vary from site to site of the lattice, one can consider several types of statistics (see, e.g., [2] for a nice review).

In particular, in the case of *kinetic* RWs the statistical weight associated with a particular path depends on both the number and type of visited lattice sites. On the other hand, one can associate the same weight K^N with each RW path having N steps, irrespective of the coordination number of visited sites. This model, known also as the *ideal chain* model, is closely related to the equilibrium statistical problem of an ideal polymer in solution. It was shown [3] that the ideal chain model and kinetic RW model in an inhomogeneous environment do not belong to the same class of universality. It is clear, on the other hand, that both statistics become equivalent on standard homogeneous lattices, and on fractal lattices having the same coordination. In view of this, a natural question arises, does a particular statistics of RW paths expected that, for large but finite b, this power law crosses to the true asymptotic law (21) for sufficiently long RW paths, $N \gg 1$ (i.e., for sufficiently small δK). This, in particular, points out the necessity of exerting much care when interpreting results of numerical simulation: for example, if the RW paths are not sufficiently long, one can easily come to an erroneous conclusion that their critical behaviour is of the power law type.

The width of the region where asymptotic behaviour (21) sets in depends on the ratio of the leading and next-to-leading term in (20). One can note that this ratio generally decreases when the lattice parameter b increases—compare the values of the coefficients λ_0, λ_1 and λ_2 for different values of b in table 1. As a consequence of this, the power law critical behaviour becomes valid over a wider and wider critical region when b grows. Finally, in the $b \to \infty$ limit the power law should be valid in the entire critical region, which corroborates our picture of crossover from localized to extended path trajectories.

Acknowledgments

This work was supported by the Serbian Ministry of Science, Technology and Development under project 1794.

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